Political Methodology II
Instrumental Variables 2

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March 13, 2022

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Experiments with Non-Compliance

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- Suppose we have a randomized controlled trial with partial non-compliance. For example, Bloom et al. (1997) evaluated an RCT commissioned by the Dept. of Labor that randomly assigned sites to participate in a job training program.
- Only about two-thirds of the enrollees in the treatment sites actually participated in the job training programs. Moreover, about 2% of the enrollees at the control sites participated in the programs anyway.
- Non-participation was not random: it was driven by lack of interest among the participants and failure of program operators to encourage participation.
- How can we make sense of the results from the experiment?

Setup

Let Z_i be a binary indicator for whether participant i was randomly assigned to participate in a job training program. Let D_i be a binary indicator for whether participant i was actually treated by participating the training program.

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We can express the actual treatment status D_i in terms of potential treatment statuses D_{0i}, D_{1i}:

$$D_i = D_{0i} + (D_{1i} - D_{0i})Z_i$$

where D_{0i} is your potential treatment status under assignment to the control group, and D_{1i} is your potential treatment status under assignment to the treatment group. We only get to observe one of these potential statuses.

Setup

With the observed values $Z_i = z$ and $D_i = d$ in hand, we can also make the following 2 $\times$ 2 table of potential outcomes, $Y(d, z)$:

	$D_i = 0$	$D_i = 1$
$Z_i = 0$	$Y_i(0,0)$	$Y_i(0,1)$
$Z_i = 1$	$Y_i(0,1)$	$Y_i(1,1)$

Four potential outcomes! Let's make some simplifying assumptions.

LATE assumptions

1. Independence: $(\{Y_i(d,z); \forall d, z\}, D_{1i}, D_{0i}) \perp\!\!\!\perp Z_i$
2. Exclusion restriction: $Y_i(d,0) = Y_i(d,1)$ for $d = 0, 1$.
3. Non-zero first stage: $\mathbb{E}[D_{1i} - D_{0i}] \neq 0$.
4. Monotonicity: $D_{1i} - D_{0i} \geq 0 \, \forall i$.

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LATE Theorem: Under assumptions 1-4,

$$\frac{\mathbb{E}[Y_i|Z_i = 1] - \mathbb{E}[Y_i|Z_i = 0]}{\mathbb{E}[D_i|Z_i = 1] - \mathbb{E}[D_i|Z_i = 0]} = \mathbb{E}[Y_{1i} - Y_{0i}|D_{1i} > D_{0i}]$$

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We call this the local average treatment for the compliers. In the context of an RCT, the numerator is known as the intent-to-treat effect (ITT) or the "reduced form", and the denominator is known as the compliance rate.

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- The first three LATE assumptions should look familiar from the constant treatment effects framework. We know why we need them for IV to work. But what is monotonicity?
- We divide the world up into four groups:
  - Always takers: People who will always select into the treatment, regardless of their assignment: $D_{0i} = D_{1i} = 1$.
  - Never takers: People who will never select into the treatment, regardless of their assignment: $D_{0i} = D_{1i} = 0$.
  - Compliers: People who will take the treatment only if they are assigned the treatment, and not take the treatment if they are not assigned: $D_{1i} > D_{0i}$.
  - Defiers: People who will not take the treatment only if they are assigned the treatment, and take the treatment only if they are not assigned: $D_{1i} < D_{0i}$.

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Monotonicity says that the randomized encouragement pushes people's treatment statuses in only one direction ($D_1 \geq D_0$). There are no defiers.

Why we need monotonicity

- Using assumptions 1-3, we can write

$$
\begin{aligned}
&\mathbb{E}[Y_i|Z_i = 1] - \mathbb{E}[Y_i|Z_i = 0] \\
&= \mathbb{E}[Y_{0i} + (Y_{1i} - Y_{0i})D_i|Z_i = 1] - \mathbb{E}[Y_{0i} + (Y_{1i} - Y_{0i})D_i|Z_i = 0] \\
&= \mathbb{E}[Y_{0i} + (Y_{1i} - Y_{0i})D_{1i}] - \mathbb{E}[Y_{0i} + (Y_{1i} - Y_{0i})D_{0i}] \\
&= \mathbb{E}[(Y_{1i} - Y_{0i})(D_{1i} - D_{0i})]
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&= \mathbb{E}[(Y_{1i} - Y_{0i})(D_{1i} - D_{0i})]
\end{aligned}
$$

- This can be split up into two expectations, one for compliers and one for defiers.

$$
\begin{aligned}
\mathbb{E}[(Y_{1i} - Y_{0i})(D_{1i} - D_{0i})] = &\underbrace{\mathbb{E}[Y_{1i} - Y_{0i}|D_{1i} > D_{0i}]P(D_{1i} > D_{0i})}_{\text{Compliers}} \\
&+ \underbrace{\mathbb{E}[Y_{1i} - Y_{0i}|D_{1i} < D_{0i}]P(D_{1i} < D_{0i})}_{\text{Defiers}}
\end{aligned}
$$

- If treatment effects are not constant, the treatment effect for defiers could cancel out the effect for compliers.

Relationship between LATE and ATT

We can write the ATT as a weighted average of the causal effects for the compliers and always takers.

$$\begin{aligned}\text{ATT} &= \mathbb{E}[Y_{1i} - Y_{0i}|D_i = 1] \\ &= \underbrace{\mathbb{E}[Y_{1i} - Y_{0i}|D_{0i} = 1]P(D_{0i} = 1|D_i = 1)}_{\text{Always Takers}} \\ &+ \underbrace{\mathbb{E}[Y_{1i} - Y_{0i}|D_{1i} > D_{0i}]P(D_{1i} > D_{0i}|D_i = 1)}_{\text{Compliers}}\end{aligned}$$

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Similarly, the ATC is a weighted average of the causal effect for the compliers and never takers, which implies the ATE is a weighted average of the causal effects across all subgroups.

This means that the LATE is not, in general, equal to the ATT or ATC, except in cases where (usually by experimental design) there can be no always takers or no never takers.

Characterizing compliers

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Characterizing compliers

- Since the compliers drive our causal effect, we often would like to know more about them. In general, can we tell who individually is a complier? No, unless we have one-sided non-compliance.
- We can, in general, calculate the size of the complier group under monotonicity and independence. Let's look at the JTPA example.

	Not Enrolled in Training	Enrolled in Training	Total
Assigned to Control	3,663	54	3,717
Assigned to Training	2,683	4,804	7,487
Total	6,346	4,858	11,204

Counting compliers

First, let's calculate the probability of participation conditional on assignment.

	Not Enrolled in Training	Enrolled in Training	Total
Assigned to Control	0.98	0.02	1.00
Assigned to Training	0.36	0.64	1.00

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We know by monotonicity that the cells contain the following subgroups.

	Not Enrolled in Training	Enrolled in Training
Assigned to Control	Compliers + Never Takers	Always Takers
Assigned to Training	Never Takers	Compliers + Always Takers

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By independence, we can assume that the proportion of any subgroup in the control group is the same and the proportion of any subgroup in the treatment group. So we can calculate the proportions of each subgroup as:

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$$P(\text{Always Taker}) = P(D = 1|Z = 0) = 0.02$$

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$$P(\text{Never Taker}) = P(D = 0|Z = 1) = 0.36$$

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P(\text{Always Taker}) &= P(D=1|Z=0) = 0.02 \\
P(\text{Never Taker}) &= P(D=0|Z=1) = 0.36 \\
P(\text{Complier}) &= P(D=0|Z=0) - P(D=0|Z=1) = 0.98 - 0.36 = 0.62 \\
&= P(D=1|Z=1) - P(D=1|Z=0) = 0.64 - 0.02 = 0.62
\end{aligned}$$

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&= P(D=1|Z=1) - P(D=1|Z=0) = 0.64 - 0.02 = 0.62
\end{aligned}
$$

Sanity Check: P(Complier) + P(Always Taker) + P(Never Taker) = 1 under monotonicity.

	Not Enrolled in Training	Enrolled in Training	Total
Assigned to Control	0.98	0.02	1.00
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Characterizing compliers

With covariates, we can do a bit better than just counting compliers. Let's start with a binary covariate X_i, such as a dummy variable for people under 30. Suppose we are interested in how much more likely compliers are to be under 30. Formally:

$$\frac{P(X_{1i} = 1 | D_{1i} > D_{0i})}{P(X_{1i} = 1)} = \frac{P(D_{1i} > D_{0i} | X_{1i} = 1)}{P(D_{1i} > D_{0i})}$$
$$= \frac{E[D_i | Z_i = 1, X_{1i} = 1] - E[D_i | Z_i = 0, X_{1i} = 1]}{E[D_i | Z_i = 1] - E[D_i | Z_i = 0]}$$

The second equality comes from Bayes' Rule, and the third equality comes from the same logic we used to count compliers previously.

Abadie Kappa

Let's say we have a continuous covariate. How can we estimate the mean for compliers? Suppose we could find a weighting function that gave us the probability unit i is a complier, conditional on X_i.

$$P(D_{1i} > D_{0i}|X_i) = 1 - \underbrace{P(D_{1i} = D_{0i} = 1|Z_i = 0, X_i)}_{\text{Always Takers}} - \underbrace{P(D_{1i} = D_{0i} = 0|Z_i = 1, X_i)}_{\text{Never Takers}}$$

$$= 1 - \frac{\mathbb{E}[D_i(1 - Z_i)|X_i]}{1 - P(Z_i = 1|X_i)} - \frac{\mathbb{E}[(1 - D_i)Z_i|X_i]}{P(Z_i = 1|X_i)}$$

$$= \mathbb{E}[\kappa_i]$$

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&= 1 - \frac{\mathbb{E}[D_i(1 - Z_i)|X_i]}{1 - P(Z_i = 1|X_i)} - \frac{\mathbb{E}[(1 - D_i)Z_i|X_i]}{P(Z_i = 1|X_i)} \\
&= \mathbb{E}[\kappa_i]
\end{aligned}
$$

So the weight at the individual level is

$$
\kappa_i = 1 - \frac{D_i(1 - Z_i)}{1 - P(Z_i = 1|X_i)} - \frac{(1 - D_i)Z_i}{P(Z_i = 1|X_i)}
$$

How to get it: estimate $P(Z_i = 1|X_i)$ using regression (e.g., linear, logit, probit), predict values for each i for the denominator, and plug in observed values for Z_i and D_i for the numerator.

Abadie Kappa

Then we can use our weight to estimate

$$\mathbb{E}[X_i|D_{1i} > D_{0i}] = \frac{\mathbb{E}[\kappa_i X_i]}{\mathbb{E}[\kappa_i]}$$

We can also use the kappa to estimate the local average response function (LARF), provided that the LATE assumptions hold conditional on X_i.

$$\mathbb{E}[g(Y_i, D_i, X_i)|D_{1i} > D_{0i}] = \frac{E[\kappa_i g(Y_i, D_i, X_i)]}{\mathbb{E}[\kappa_i]}$$

where $g(Y_i, D_i, X_i)$ is any measurable function of (Y_i, D_i, X_i) with a finite expectation. Special case: When $P(Z_i = 1|X_i)$ is estimated with linear regression, the kappa-weighted linear regression for $E[Y_i|D_i, X_i, D_{1i} > D_{0i}]$ is equal to 2SLS with covariates.