MLE

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Data: $\{w_i : i = 1, \cdots, n\}$ iid, $w_i \sim f(w,\theta), \theta \in \Theta \in \mathbb{R}_k$

Because of independence:

Likelihood Function.

$$L_n(\theta) = \prod_{i=1}^{n} f(w_i,\theta)$$

$$logL_n(\theta) = \frac{1}{n}\sum_{i=1}^{n} logf(w_i,\theta)$$

MLE Estimator

$$\hat{\theta} = \text{arg max}_{\theta\in\Theta}\frac{1}{n}\sum_{i=1}^{n} logf(w_i,\theta)$$

Information Matrix

$$\Omega = \mathbb{E}\frac{\partial logf(w_i,\theta_0)}{\partial\theta}\frac{\partial logf(w_i,\theta_0)}{\partial\theta'} = -B = I(\theta_0)$$

Asymptotic Convergence

$$\sqrt{n}(\hat{\theta}-\theta_0) \to_d N(0, B^{-1}\Omega B^{-1}) = N(0, [I(\theta_0)]^{-1})$$

Binary Choice.

$$\mathbb{P}(Y_i = 1|X_i) = 1 - \mathbb{F}(-X_i'\beta)$$

- Probit: $F(u) = \Phi(n)$ - Standard normal $N(0,1)$ CDF
- Logit:
  - $F(u) = \Lambda(u) = \frac{1}{1+e^{-u}}$
  - $f(u) = \Lambda'(u) = (1-e^{-u})^{-2}e^{-u}$

General case: maximize the following Log Likelihood

$$logL_n(\beta) = \frac{1}{n}\sum Y_i log\mathbb{F}(X_i'\beta)\frac{1}{n}\sum(1-Y_i)log(1-\mathbb{F}(X_i'\beta))$$

Solutions for Logit :

$$log(\frac{p}{1-p}) = \beta X$$

Point Estimate

$$\mathbb{E}(Y_i|X_i) = \mathbb{P}(Y_i = 1|X_i) = \Lambda(X_i'\beta)$$

Variance

$$I(\beta) = \mathbb{E}[(\Lambda(X_i'\beta)(1 - \Lambda(X_i'\beta))X_iX_i']$$

Marginal Effects

$$\frac{\partial P(Y_i = 1|X_i)}{\partial X_i} = f(X_i'\beta)\beta = \Lambda(X_i'\beta)(1 - \Lambda(X_i'\beta))\beta$$