MATHEMATICAL STATISTICS

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Moments of Random Variables. Random variable x with support $[\underline{x}, \bar{x}]$

N-th moment $= \mathbb{E}x^n$

$$\mathbb{E}(X) = \int_{\underline{x}}^{\bar{x}} xf(x)dx$$

$$V(X) = \int_{\underline{x}}^{\bar{x}} (X - \mathbb{E}(X))^2 dx = \mathbb{E}[(X - \mathbb{E}X)^2] = \mathbb{E}(X^2) - (\mathbb{E}X)^2$$

For k-vectors

$$V(X) = \mathbb{E}(XX') - \mathbb{E}(X)\mathbb{E}(X)'$$
$$Cov(X,Y) = \mathbb{E}[(X - \mathbb{E}X)(Y - \mathbb{E}Y)']$$

Moment Generating Function

$$M^{(s)}(t) = \int_{-\infty}^{\infty} x^s e^{tx} dx$$

Standard Normal: $e^{t^2/2}$

Correlation Coefficient

$$\frac{Cov(x,y)}{\sigma_x \sigma_y}$$

Bias-variance Tradeoff

$$MSE(\hat{\theta}) = \mathbb{E}((\hat{\theta} - \theta)^2) = \mathbb{E}(\hat{\theta} - \mathbb{E}\hat{\theta})^2 + [\mathbb{E}(\hat{\theta} - \theta)]^2 = \mathbb{V}(\hat{\theta}) + \mathbb{B}(\hat{\theta})^2$$

Hypothesis testing. **Two-sided** Confidence interval

$$CI_{1-\alpha} : \{\hat{\beta} \in \mathbb{R}_k : P(\beta \in CI_{1-\alpha}) = 1 - \alpha\} = [\hat{\theta} - \omega z_{1-\alpha/2}, \hat{\theta} + \omega z_{1-\alpha/2}]$$

Test statistic

$$s = \left| \frac{\hat{\theta} - \theta_0}{\omega} \right| \leq z_{1-\alpha/2}$$

where $\omega = \sqrt{\sigma^2/n} = \frac{1}{n-k} \sum \hat{u_i}^2$

P-value $= [1 - \Phi(|s|)] \times 2$

Power $=$ Pr(reject H_0 | H_1 is true) : $\pi(\theta) = P[|z + s| > z_{1-\alpha/2}]$

Cauchy Schwartz Inequality: $Cov(X,Y)^2 \leq \sigma_x \sigma_y$

Markov Inequality: $P[|X| > \epsilon] \leq E[|X|^r]/\epsilon^r$

Distributions.

- Standard normal : $z \sim (0,1); \mu = 0, \sigma = 1$
- Chi-squared : $\chi^2_n = \sum^n z_i^2, \mu = n, \sigma = 2n$
- t : let $x \sim \chi^2_n, Y = z/\sqrt{x/n} \to y \sim t_n$
- F : let $x_1 \sim \chi^2_{n1}$, and $x_2 \sim \chi^2_{n2}$; $y = \frac{x_1/n_1}{x_2/n_2} \to Y \sim Fn_1, n_2$

Asymptotics. $\to_p$ - Convergence in Probability $\to_d$ - Convergence in Distribution

- Law of Large Numbers: $X_1, \ldots, X_n$ are IID; $\mathbb{E}[X_1] < \infty \Rightarrow n^{-1}\sum^n X_i \to_p \mathbb{E}X_1$ as $n \to \infty$
- Cramer Convergence : $X_n \to_d X; Y_n \to_p c \Rightarrow$
  - $X_n + Y_n \to_d X + c$
  - $X_n Y_n \to_d cX$
  - $X_n / Y_n \to_d X/c$
- Slutsky's Theorem: $X_n \to_p X; h(.)$ is continuous $\Rightarrow; h(X_n) \to_p h(x)$
- Central Limit Theorem: $X_1, \ldots, X_n$ are IID; $\mathbb{E}[X_1] = 0; 0 < \mathbb{E}X_1^2 < \infty \Rightarrow \sqrt{n}\sum^n X_i \to_d N(0, \mathbb{E}X^2)$ as $n \to \infty$
- Continuous Mapping Theorem: $X_n \to_d X; h(.)$ is continuous $\Rightarrow; h(X_n) \to_d h(x)$
- Delta Method: $\sqrt{n}(\hat{\theta} - \theta) \to_d Y \Rightarrow \sqrt{n}(h(\hat{\theta}) - h(\theta)) \to_d [\partial h(\theta)/\partial\theta']Y$