

Formalising QWERTYnomics

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Reasoning LLMs do reasonably well at building simple applied economic theory models.

Opening Prompt (with David 1985 paper attachment; familiarity with Carlsson and Van Damme presumed of the LLM):

I've attached a classic paper by Paul David on the explanations for persistence in technology, with the leading example of the QWERTY keyboard, which has continued to dominate despite being strictly worse than DVORAK and other keyboard layouts along many dimensions. David mentions three key features: "technical interrelatedness", "economies of scale", and "quasi-irreversibility of investment" as ingredients of "QWERTYnomics". The model is entirely verbal, but extremely insightful. In the game theory literature since David's article, Global Games have become one of the dominant modelling devices for such situations where equilibrium selection is a key ingredient. Uncertain economic fundamentals are summarized by a state θ and each player observes a different signal of the state with a small amount of noise. In the original Carlsson and van Damme investment game, there are two actions: invest and not invest, with payoff matrix

(Invest, NotInvest)
Invest, (θ , θ), ($\theta - 1$, 0)
NotInvest, (0, $\theta - 1$), (0, 0)

[(row player payoff, column player payoff] with θ unknown (or observed with noise: $\text{signal} = \theta + \text{noise}$). Come up with a version of a game like this that formalizes some part of QWERTYnomics [this captures the interrelatedness and economies of scale piece, at first cut]. Write parsimoniously and rigorously.

Followed up by some corrections and prodding.

1 Claude Opus 4: A Global Game Model of Keyboard Standard Adoption

Consider a continuum of agents $i \in [0, 1]$ (typists, firms, or typewriter manufacturers) who must choose between two keyboard standards: QWERTY (Q) and an alternative like Dvorak (D). Let θ represent the fundamental efficiency advantage of Dvorak over QWERTY.

1.0.1 Payoff Structure

Agent i 's payoff from choosing action $a_i \in \{Q, D\}$ when proportion ℓ of other agents choose D is:

$$u_i(a_i, \ell, \theta) = \begin{cases} \theta + g(\ell) - c & \text{if } a_i = D \\ g(1 - \ell) & \text{if } a_i = Q \end{cases}$$

where:

- θ is the intrinsic efficiency gain from Dvorak (e.g., typing speed improvement)
- $c > 0$ is the switching/training cost for adopting Dvorak
- $g(\cdot)$ captures network effects with $g'(\cdot) > 0$, $g''(\cdot) > 0$, and $g(0) = 0$

The function $g(\ell)$ represents technical interrelatedness—the value of using the same standard as others due to:

- Compatibility of trained typists across firms
- Availability of replacement parts and repair services
- Standardization of typing instruction

1.0.2 Information Structure

The true state θ is drawn from an improper uniform prior on \mathbb{R} . Each agent i observes a private signal:

$$x_i = \theta + \sigma \epsilon_i$$

where $\epsilon_i \sim N(0, 1)$ i.i.d. across agents, and $\sigma > 0$ is small.

1.0.3 Equilibrium Analysis

An agent chooses D if and only if:

$$\theta + g(\ell) - c \geq g(1 - \ell)$$

This simplifies to:

$$\theta \geq c - g(\ell) + g(1 - \ell) \equiv \hat{\theta}(\ell)$$

Note that $\hat{\theta}'(\ell) = -g'(\ell) - g'(1 - \ell) < 0$, creating strategic complementarity.

In the limit as $\sigma \rightarrow 0$, there exists a unique threshold equilibrium where agents follow a cutoff strategy: choose D if and only if $x_i \geq x^*$. The equilibrium cutoff satisfies:

$$x^* = \hat{\theta}(\Phi((x^* - \theta)/\sigma))$$

where Φ is the standard normal CDF.

An agent chooses D if:

$$\theta + g(\ell) - c \geq g(1 - \ell)$$

In equilibrium with threshold strategy x^* , the proportion choosing D is:

$$\ell^* = \Pr(x_i \geq x^*) = \Pr(\theta + \sigma \epsilon_i \geq x^*) = 1 - \Phi\left(\frac{x^* - \theta}{\sigma}\right)$$

At the threshold, an agent with signal x^* must be indifferent:

$$x^* + g(\ell^*) - c = g(1 - \ell^*)$$

Substituting the expression for ℓ^* :

$$x^* = c - g\left(1 - \Phi\left(\frac{x^* - \theta}{\sigma}\right)\right) + g\left(\Phi\left(\frac{x^* - \theta}{\sigma}\right)\right)$$

This is a fixed-point equation where **the shape of $g(\cdot)$ matters**. The derivatives $g'(\cdot)$ determine:

- The slope of the best response function
- The strength of strategic complementarities
- Whether multiple equilibria exist

1.0.4 Why Network Effects Shape Matters

Consider two cases:

1. **Linear network effects:** $g(\ell) = \gamma\ell$
 - Threshold: $x^* = c - \gamma(1 - 2\ell^*)$
 - Single equilibrium for small σ
2. **Convex network effects:** $g(\ell) = \gamma\ell^2$
 - Threshold: $x^* = c - \gamma[(1 - \ell^*)^2 - (\ell^*)^2]$
 - Potentially multiple equilibria even for small σ if γ is large

The convexity of $g(\cdot)$ creates stronger “tipping point” dynamics—small changes in beliefs can trigger large shifts in adoption.

1.0.5 QWERTY Lock-in

The model captures QWERTY persistence through:

1. **High switching costs:** When c is large relative to θ , even an intrinsically superior standard (Dvorak) may not be adopted.
2. **Network effects:** The convexity of $g(\cdot)$ means early adoption advantages compound—once QWERTY achieves critical mass, the coordination benefits dominate efficiency gains.
3. **Quasi-irreversibility:** Historical accidents determining initial conditions (which standard achieves early adoption) have permanent effects through the self-reinforcing dynamics of $g(\cdot)$.

This formalization shows how small initial advantages, amplified by network effects and switching costs, can lead to permanent dominance of an inferior standard—the essence of QWERTYnomics.

1.1 Gemini’s review

The provided alternative formalization is a well-structured global game model that effectively captures the essence of “QWERTYnomics.” Let’s evaluate it and compare it to the previous formalization.

Evaluation of the Provided Formalization:

This model defines agent payoffs as:

- $u_i(D, l, \theta) = \theta + g(l) - c$ (if adopting Dvorak, where l is the proportion of others adopting Dvorak) [cite: 3]
- $u_i(Q, l, \theta) = g(1 - l)$ (if sticking with QWERTY) [cite: 3]

Key components include:

- θ : The intrinsic efficiency gain from Dvorak. [cite: 3]
- c : A switching/training cost for adopting Dvorak. [cite: 3]
- $g(\cdot)$: A function capturing network effects, with $g'(\cdot) > 0$ (more users of a standard increase its value), $g''(\cdot) > 0$ (convexity, implying increasing marginal network benefits or “critical mass” effects), and $g(0) = 0$ (no network benefit if no one else uses the standard). [cite: 3] This function explicitly models “technical interrelatedness.” [cite: 3]

Strengths:

1. **Clear Separation of Effects:** The model clearly distinguishes between intrinsic efficiency (θ), direct switching costs (c), and network externalities ($g(\cdot)$). [cite: 3] This allows for a nuanced analysis of how each component contributes to adoption decisions.
2. **Explicit Network Function:** The use of a general function $g(\cdot)$ with defined properties ($g' > 0, g'' > 0$) is a strong feature. [cite: 3] The convexity ($g'' > 0$) is particularly good for modeling how “early adoption advantages compound” [cite: 9], a key aspect of path dependence.
3. **Strategic Complementarity:** The model correctly identifies that the threshold for adopting Dvorak, $\hat{\theta}(l) = c - g(l) + g(1 - l)$, decreases as more agents adopt Dvorak (i.e., $\hat{\theta}'(l) < 0$) [cite: 5]. This captures the strategic complementarity inherent in standards adoption.

4. QWERTYnomics Elements:

- **Technical Interrelatedness:** Directly modeled by $g(\cdot)$, representing “the value of using the same standard as others.” [cite: 3]
- **Economies of Scale:** Captured by $g'(\cdot) > 0$ and the convexity $g''(\cdot) > 0$. [cite: 3, 9]
- **Quasi-irreversibility:** Represented by the switching cost c [cite: 3] and the self-reinforcing dynamics of $g(\cdot)$ that can make historically determined initial conditions permanent. [cite: 10]

5. **Standard Global Game Machinery:** It employs the typical information structure ($x_i = \theta + \sigma\epsilon_i$) and points to a unique threshold equilibrium in the limit of small noise. [cite: 4, 7]

How it Compares to My Previous Formalization (Carlsson & van Damme style):

My previous model used the following payoff structure (D=DVORAK/Invest, Q=QWERTY/Not Invest):

- $U(D, D) = \theta_{mine}$
- $U(Q, Q) = 0$
- $U(D, Q) = \theta_{mine} - 1$ (player adopts D, others Q)
- $U(Q, D) = 0$ (player adopts Q, others D)

Similarities:

- Both are global game models aimed at explaining how an inferior standard can persist due to coordination failure and network effects.
- Both model a fundamental parameter related to Dvorak’s superiority (θ_{mine} vs. θ in the provided model).
- Both include a penalty for unilateral adoption of the new standard (the “-1” cost in my model; the “-c” and potentially lower $g(l)$ vs. $g(1 - l)$ in the provided model).
- Both predict a threshold-based equilibrium.

Differences and Relative Advantages:

1. Generality of Network Effects:

- **Provided Model:** More general and explicit. The $g(\cdot)$ function allows for flexible modeling of how network benefits accrue, including critical mass effects via $g'' > 0$. [cite: 3, 9]
- **My Model:** Simpler, with network effects being somewhat binary (full benefit if coordinated, a fixed penalty if not). It’s a specific case that could be represented by particular choices of $g(\cdot)$ and c in the provided model’s framework.

2. Clarity of Parameters:

- **Provided Model:** Offers a cleaner separation. θ is purely intrinsic efficiency, c is purely switching cost, and $g(\cdot)$ handles all network aspects. [cite: 3]
- **My Model:** θ_{mine} is the benefit of Dvorak *when universally adopted* (so it includes Dvorak’s network effect at $l = 1$). The “-1” is a composite penalty for being misaligned (incorporating both switching costs and forgone network benefits).

3. Baseline and Normalization:

- **Provided Model:** $g(0) = 0$ means no network benefit if you are alone on a standard. [cite: 3] If everyone uses QWERTY ($l = 0$), a QWERTY user gets $g(1)$. If everyone uses Dvorak ($l = 1$), a Dvorak user gets $\theta + g(1) - c$.
- **My Model:** (Q,Q) payoff is normalized to 0. This means θ_{mine} is implicitly relative to the fully established QWERTY network benefit.

4. Explicit Addressing of QWERTYnomics:

- The provided model’s paper explicitly states how c , $g(\cdot)$, and its convexity map to high switching costs, network effects, and quasi-irreversibility, linking them directly to QWERTY persistence. [cite: 8, 9, 10] My verbal explanation did this, but the provided model’s structure makes these links very direct within the mathematical formulation itself.

Conclusion of Comparison:

The provided formalization is more general and arguably more explicit in its components than my Carlsson & van Damme-style model. Its strength lies in the flexible $g(\cdot)$ function which can be tailored to different assumptions about how network externalities manifest, especially the notion of accelerating benefits ($g'' > 0$). This makes it very well-suited to capture the “lock-in” dynamics David describes.

My model is simpler and more directly uses a canonical game structure. While it captures the core ideas, the provided model offers a more detailed and decomposable representation of the economic forces at play. Both successfully demonstrate how coordination challenges can lead to the persistence of an inefficient standard, which is the central theme of QWERTYnomics. The provided model does an excellent job of rigorously formalizing David’s verbal arguments.

2 Gemini 2.5 Pro: A Global Game Model of Keyboard Adoption

Consider a continuum of economic agents, each facing a decision to either **Adopt DVORAK (D)**, a potentially more efficient keyboard layout, or **Stick with QWERTY (Q)**, the established standard. The QWERTY standard’s persistence, despite alternatives like DVORAK offering potential efficiency gains (e.g., 20-40% faster typing as noted by Apple advertising for DSK, or the U.S. Navy’s findings on retraining amortization [cite: 19, 22]), is the phenomenon we aim to model.

Payoffs: The payoff to an agent depends on their own action and the actions of other agents, reflecting network externalities. We adapt the Carlsson and van Damme investment game structure:

- If an agent adopts DVORAK and other agents also adopt DVORAK, the payoff is θ .
- If an agent sticks with QWERTY and other agents also stick with QWERTY, the payoff is 0. This normalizes the value of the established QWERTY network.
- If an agent adopts DVORAK, while other agents stick with QWERTY, the DVORAK adopter’s payoff is $\theta - 1$. The “1” represents the cost of unilateral adoption (e.g., retraining, incompatibility, lack of software/hardware support).
- If an agent sticks with QWERTY, while other agents adopt DVORAK, the QWERTY user’s payoff is 0. (In the symmetric C&vD game, the other player receives $\theta - 1$; here, the QWERTY user is simply left out of the DVORAK benefits/costs if they don’t switch).

The payoff matrix for a representative agent i , based on their action and the action of a representative other agent j , is:

	Adopt DVORAK	Keep QWERTY
Adopt DVORAK	θ, θ	$\theta - 1, 0$
Keep QWERTY	$0, \theta - 1$	$0, 0$

Interpretation of Parameters in “QWERTYnomics”:

1. **θ (State Variable):** Represents the **net fundamental benefit of DVORAK if it were to become the universally adopted standard**, relative to QWERTY. This benefit includes DVORAK’s purported higher intrinsic efficiency. [cite: 18, 19] A higher θ signifies a greater advantage for DVORAK.
2. **Technical Interrelatedness and Economies of Scale:** [cite: 65, 67, 73] These are captured by how payoffs change with others’ actions.
 - The payoff θ for (D, D) reflects the full realization of DVORAK’s benefits when there is system-wide compatibility (hardware and trained typists). [cite: 67]
 - The payoff 0 for (Q, Q) reflects the existing network benefits of the entrenched QWERTY standard.
 - The difference in payoff when an agent is coordinated with others versus uncoordinated (e.g., θ for DVORAK adoption vs. $\theta - 1$) highlights the scale economies: the technology is more valuable the more users adopt it. David notes that the overall user costs of a system tend to decrease as it gains acceptance. [cite: 71]
3. **Quasi-irreversibility of Investment:** [cite: 65, 91] This is primarily captured by the **cost term “-1”** in the payoff $\theta - 1$ for an agent who unilaterally adopts DVORAK.
 - This cost signifies the difficulties and expenses an early adopter faces due to the prevalence of the QWERTY standard. These include the costs of retraining oneself in DVORAK typing skills (the “software conversion” [cite: 91, 92]), finding compatible typewriters or software, and operating in an environment largely geared towards QWERTY. David notes “the high costs of software ‘conversion’ and the resulting quasi-irreversibility of investments in specific touch-typing skills.” [cite: 91] Even if DVORAK is superior (i.e., $\theta > 0$), if this superiority is not enough to overcome the initial adoption cost (i.e., if $\theta < 1$), individuals may not switch.

Global Game Structure: Agents do not know the true value of θ . Instead, each agent i observes a private noisy signal: $x_i = \theta + \epsilon_i$, where ϵ_i is a small, idiosyncratic noise term (e.g., drawn from a normal distribution with mean 0).

Let k be the proportion of other agents who choose to Adopt DVORAK. An agent i ’s expected payoff from adopting DVORAK is $k\theta + (1 - k)(\theta - 1) = \theta - (1 - k)$. The payoff from sticking with QWERTY is $k(0) + (1 - k)(0) = 0$. Thus, agent i will choose to Adopt DVORAK if their expectation of $\theta - (1 - k)$ is greater than 0. Given their private signal x_i , they will adopt DVORAK if $E[\theta|x_i] > 1 - E[k|x_i]$, pivotal reasoning].

In typical global game models of this type, agents adopt a threshold strategy: agent i adopts DVORAK if $x_i > x^*$, and sticks with QWERTY if $x_i < x^*$. The noise term ϵ_i ensures that the aggregate adoption level k is a continuous, increasing function of θ , which often leads to a unique equilibrium threshold x^* .

An agent adopts DVORAK if $E[\theta|x_i] > 1 - E[k|x_i]$. Here’s how the noise comes in:

- Agents use a threshold strategy: adopt DVORAK if their private signal $x_i > x^*$.
- The proportion of agents k adopting DVORAK, when the true state is θ , is $k = P(x_j > x^* | \theta)$. Given $x_j = \theta + \epsilon_j$, this becomes $k = P(\epsilon_j > x^* - \theta) = 1 - F_\epsilon(x^* - \theta)$, where F_ϵ is the cumulative distribution function (CDF) of the noise ϵ_j .
- In many global game settings (particularly with an improper prior on θ or when analyzing the critical threshold for coordination as noise vanishes), the pivotal agent who observes $x_i = x^*$ believes that $\theta = x^*$. At this point, the proportion of agents switching is $k^* = 1 - F_\epsilon(x^* - x^*) = 1 - F_\epsilon(0)$.
- The equilibrium threshold x^* is then found by solving $x^* = 1 - k^*$, which simplifies to $x^* = 1 - (1 - F_\epsilon(0)) = F_\epsilon(0)$.
- If the noise ϵ_j is symmetrically distributed around 0 (e.g., $\epsilon_j \sim N(0, \sigma^2)$), then $F_\epsilon(0) = 0.5$. Thus, the unique threshold selected by the global game is $x^* = 0.5$.

So, the characteristics of the noise distribution (specifically, its median through $F_\epsilon(0)$) are indeed crucial in determining the exact value of the switching threshold x^* .

Implications for QWERTY's Persistence:

- If x^* is the equilibrium threshold, QWERTY persists if the true θ (or agents' perception of it based on their signals) falls below x^* .
- The model demonstrates how an inefficient standard (QWERTY, yielding payoff 0) can remain locked-in [cite: 64] even when a superior alternative exists (DVORAK, potentially offering payoff $\theta > 0$). This occurs if $\theta < 1$ (making unilateral adoption costly) and agents are pessimistic about others' adoption (expecting low k), or if uncertainty (the noise ϵ_i) and coordination problems prevent a mass shift.
- The "historical accidents" [cite: 4, 84] that allowed QWERTY to gain an early, albeit slender, lead [cite: 46, 48, 89] established the conditions (the "1" in $\theta - 1$) making a switch to DVORAK difficult, effectively setting a higher bar for θ to trigger a transition.

2.1 Claude's review

This alternative formalization is excellent and has several strengths compared to mine. Let me evaluate it systematically:

2.1.1 Strengths of the Alternative Model

1. **Cleaner Payoff Structure:** The alternative uses the exact Carlsson-van Damme payoff matrix, making it more directly connected to the established global games literature. This is more elegant than my additive formulation with separate network effects function $g(\cdot)$.
2. **Better Microfoundation:** By explicitly modeling the payoff matrix for pairwise interactions and then aggregating over the population (expected payoff $= k\theta + (1 - k)(\theta - 1) = \theta - (1 - k)$), it provides clearer microfoundations. My model jumped directly to aggregate payoffs without this intermediate step.
3. **Unified Cost Interpretation:** The "-1" term elegantly captures both switching costs and lack of network benefits simultaneously. In my model, I separated these into c (switching cost) and $g(\cdot)$ (network effects), which is less parsimonious.
4. **Direct Literature Connection:** The model more faithfully follows the canonical global games framework, making it easier to apply known results about uniqueness and equilibrium characterization.

2.1.2 Comparative Advantages of My Model

1. **Explicit Network Effects:** My formulation with $g(\cdot)$ allows for analyzing how the **shape** of network effects (convexity, increasing returns) affects outcomes. The alternative model implicitly assumes linear network effects.
2. **Asymmetric Treatment:** My model explicitly allows QWERTY and DVORAK to have different network effect functions, which could capture historical path dependence more richly.
3. **Continuous Strategy Space Extension:** My framework more naturally extends to cases where agents might partially adopt or gradually transition between standards.

2.1.3 Technical Comparison

Both models yield similar equilibrium conditions:

- **Alternative:** Adopt DVORAK if $E[\theta | x_i] > 1 - E[k | x_i]$
- **Mine:** Adopt DVORAK if $\theta \geq c - g(\ell) + g(1 - \ell)$

With linear network effects ($g(\ell) = \ell$), my condition becomes $\theta \geq c - 2\ell + 1$, which is equivalent to the alternative's with $c = 1$.

2.1.4 Overall Assessment

The alternative formalization is **superior for this specific application** because:

1. It maintains closer fidelity to both the Carlsson-van Damme framework and David’s original narrative
2. It achieves the same insights with fewer parameters
3. The interpretation is more transparent—the “-1” directly represents the disadvantage of being a technological orphan

My model would be preferable only if one needed to:

- Analyze non-linear network effects explicitly
- Model asymmetric standards with different network properties
- Consider gradual adoption or mixed strategies

For capturing the essence of QWERTYnomics in the most parsimonious way while maintaining rigorous game-theoretic foundations, the alternative model is the better choice.